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OF A TETHERING CABLE

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The optimum configuration of a tethering cable is determined. The optimum condition is defined as the cable configuration which results in the maximum depth for a given cable tension. The calculations presented in the report are for cable profile and operating conditions for which the profile pressure drag is negligible compared to the friction drag. The results offer a rapid determination of the optimum cable configuration for all operating conditions that satisfy the assumptions used in determining the cable loading functions. The same optimizing procedure could be used to determine the optimum configuration of any cable by using the loading functions applicable to the particular cable and operating conditions. It is found that for the optimum configuration of heavy cables $(\frac{W}{R} \geq 0)$, the submerged body is located forward of the surface vessel.

Introduction

Experience with towing cables has shown that the depth attainable decreases rapidly as the towing velocity increases. Reducing the drag of the cable improves the situation somewhat, but extreme depths are possible only at quite low towing velocities. It has been proposed, therefore, to use a tethering system in which a self-propelled body is interconnected to the towing vehicle by a cable. The cable is used to transmit the propulsion power to the submerged body and is also used to transmit control and data signals between the two vehicles. It would be expected that for a given cable tension and towing velocity, the tethering system should increase the depth attainable to approximately twice that of the towing system.

Initial calculations have shown that if the end points of the cable are chosen there are an infinite number of cable configurations which will satisfy the choice of end points. The maximum cable tension is different for each configuration and there is an optimum configuration for which the cable tension is a minimum.

It is the purpose of this report to determine the optimum cable configuration for any given operating condition. The optimum condition is defined as the cable configuration which will maximize the depth for a given cable tension. The end points of the cable are not chosen but rather are determined by the optimum configuration. The results show that the optimum position of the submerged body is forward of the surface vessel for "heavy" cables.

The results are general and may be applied to any operating conditions which do not violate the assumptions used in determining the set of cable loading functions used in the evaluation of the cable equations.

The choice of the loading functions used in the present calculations was based on the type of cable and operating conditions that were of interest for this particular project. Obviously, for cable profiles and operating conditions that are not satisfied by the loading functions used in this report, the same optimizing procedure could be performed with evaluations based on loading functions that best describe the particular cable profile and operating conditions under consideration.

The various loading functions that have been proposed and their range of applicability are adequately discussed in References 2 and 3.

1. Theory.

A. Cable Equations.

The forces acting on a cable element operating at the conditions assumed for this project are shown in Fig. 1. Resolving the forces into components acting normal and tangential to the cable element, the following equations may be written,

$$\frac{dT}{ds} = R\cos\theta + w\sin\theta \tag{1}$$

$$\frac{d\theta}{ds} = \frac{w\cos\theta - R\sin\theta}{T} \tag{2}$$

Eliminating ds from the above expressions yields

$$\frac{dT}{T} = \frac{R\cos\theta + w\sin\theta}{w\cos\theta - R\sin\theta}\, d\theta \tag{3}$$

Integrating equation (3)

$$\tau = \frac{T}{T_o} = e^{\int_{\theta_o}^{\theta} \frac{R\cos\theta + w\sin\theta}{w\cos\theta - R\sin\theta}\, d\theta} \tag{4}$$

Substituting (4) into (2) and integrating yields

$$s = T_o \int_{\theta_o}^{\theta} \frac{\tau}{w\cos\theta - R\sin\theta}\, d\theta \tag{5}$$

and by the geometry of the problem

$$dx = \cos\theta\, ds$$

$$dz = \sin\theta\, ds \tag{6}$$

$$x = T_o \int_{\theta_o}^{\theta} \frac{\tau \cos\theta\, d\theta}{w\cos\theta - R\sin\theta} \tag{7}$$

$$z = T_o \int_{\theta_o}^{\theta} \frac{\tau \sin\theta\, d\theta}{w\cos\theta - R\sin\theta} \tag{8}$$

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The evaluation of equations (4), (5), (7), and (8) depend on the assumed relationship between R and θ.

B. Cable Loading Functions.

The cable loading functions are defined as follows:

$$f(\theta) = \text{normal loading functions} = \frac{R \sin\theta}{R^*}$$

$$g(\theta) = \text{tangential loading functions} = \frac{R \cos\theta}{R^*}$$

where R^* is the fluid dynamic force acting on a unit length of cable when $\theta = \frac{\pi}{2}$.

For the case of a hydrodynamically clean faired cable cross section at supercritical Reynolds Number, the pressure drag should be negligible compared to the skin friction drag. The resulting fluid dynamic force is then parallel to the free stream velocity direction.

Defining the drag coefficient based on the projected area normal to the chord, as is usual in aerodynamics.

$$R^* = C_D^* \rho/_2 U^2 c^* ds$$

$$R = C_D \rho/_2 U^2 C \sin\theta ds = C_D \rho/_2 U^2 c^* ds$$

(9)

It is seen that the projected area for a length ds remains unchanged but the apparent chord of the profile is a function of θ. If it is assumed that the profile drag coefficient will change with Reynolds Number (based on chord length) in the same manner as the turbulent skin friction coefficient for $Re_x < 3 \times 10^{-7}$, for a flat plate, and if it is further assumed that the apparent change in thickness distribution will have a negligible effect on the drag coefficient, then

$$\frac{R}{R^*} = \sin^{1/5}\theta$$

(10)

The loading functions then become

$$f(\theta) = \sin^{6/5}\theta$$

$$g(\theta) = \sin^{1/5}\theta \cos\theta \tag{11}$$

A comparison of the above loading functions with those proposed by Eames[2] is given in Ref. 1. Also, it may be noted that the above loading functions have characteristics that are similar to the loading functions calculated from experimental wing data by Clark.[3]

With the definitions

$$\tau = \frac{T}{T_o} \qquad\qquad \xi = \frac{xR^*}{T_o}$$

$$\sigma = \frac{sR^*}{T_o} \qquad\qquad \eta = \frac{zR^*}{T_o} \tag{12}$$

substituting the above loading functions into equations (4), (5), (7) and (8) yields

$$\tau = c \int_{\theta_o}^{\theta} \frac{\sin^{1/5}\theta\cos\theta + \frac{w}{R^*}\sin\theta}{\frac{w}{R^*}\cos\theta - \sin^{6/5}\theta}\, d\theta \tag{13}$$

$$\sigma = \int_{\theta_o}^{\theta} \frac{\tau\, d\theta}{\frac{w}{R^*}\cos\theta - \sin^{6/5}\theta} \tag{14}$$

$$\xi = \int_{\theta_o}^{\theta} \frac{\tau\cos\theta\, d\theta}{\frac{w}{R^*}\cos\theta - \sin^{6/5}\theta} \tag{15}$$

$$\eta = \int_{\theta_o}^{\theta} \frac{\tau\sin\theta\, d\theta}{\frac{w}{R^*}\cos\theta - \sin^{6/5}\theta} \tag{16}$$

Unfortunately, the above equations cannot be solved in terms of known tabulated functions and recourse must be made either to hand or machine numerical

evaluation.

2. Evaluation for the Optimum Cable Configuration.

The optimum cable configuration is defined as the configuration which will result in the maximum depth of submergence for a given cable tension, or conversely as the configuration which will result in the minimum tension for a given depth of submergence.

The cable operating conditions and the cable profile determine the parameters of the problem.

$$\frac{w}{R^*} = \frac{(\rho_c - \rho_f) \, a \, C \, t \, g}{C_D^* \rho_f / 2U^2 C} \tag{17}$$

where a is a geometric parameter relating the cross section area of the profile to the cross section area of the circumscribed rectangle. The value of a may be approximated by assuming the profile has a cross section area equal to an ellipse of major axis C and minor axis t.

$$a = \frac{\pi}{4}$$

For most profiles, a will be slightly less than this approximate value.

Equation (17) then reduces to

$$\frac{w}{R^*} = \frac{\pi}{4C_D^*} \frac{(\gamma_c - \gamma_f)t}{P_D} \tag{18}$$

or

$$\frac{w}{R^*} = \frac{\pi}{2C_D^*} \left(\frac{\rho_c}{\rho_f} - 1\right) \frac{gt}{U^2}$$

Considering equation (16), there are a number of sets of θ_o, θ which will result in a given value for η but one particular set should result in a minimum cable tension. It is certain that as θ_o decreases, θ must also decrease if η is to remain constant. Also, by equation (13) the tension in the cable is equal to T_o at θ_o, and decreases as θ decreases to the point on the cable where $(\sin^{1/5}\theta\cos\theta + \frac{w}{R_*}\sin\theta)$ is equal to zero and thereafter the cable tension increases as θ decreases. It is clear then that for the tethering cable the higher tensions occur at the end points of the cable. Furthermore, if the end point tensions are to be a minimum for a given η, they must be equal. The optimum cable configuration is determined then by the conditions $\tau = 1$, $\eta = \eta_{max}$.

Equations (14), (15) and (16) were evaluated for $\tau = 1$ and with θ_o and $\frac{w}{R_*}$ as parameters.

3. Results.

Fig. 2 is a plot of the results obtained for η. It is seen that η goes through a maximum and the value of θ_o thus determined is used to determine the remainder of the cable parameters for the optimum configuration.

Fig. 3 is a plot of all the cable parameters for the optimum configuration. The geometric parameters are defined in Fig. 4.

4. Sample Calculations.

The following information must be given:

$$C_D^*, \ \rho_c, \ \rho_f, \ t, \ U, \ T_{max}, \ C^*$$

For the purposes of an example, the following data is used for a cable in sea water.

$$C_D^* = .006$$

$$\rho_c/\rho_f = 1.2$$

$$t = 1/12 \ \text{foot}$$

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$$U = 40 \text{ ft/sec}$$

$$T_{max} = 10{,}000 \text{ pounds}$$

$$c^* = 0.5 \text{ ft.}$$

Using equation (18),

$$\frac{w}{R^*} = \frac{\pi}{2(.006)} \cdot (1.301) \left[\frac{32.2 \cdot \frac{1}{12}}{(40)^2}\right]$$

$$\frac{w}{R^*} = .1310$$

By Fig. 4, the optimum configuration is given by

$$\theta_o = 156.5^{\circ}$$

$$\theta_s = 39.2^{\circ}$$

$$\alpha = 8.9^{\circ}$$

and the cable dimensionless parameters are

$$\eta = 1.363$$

$$\sigma = 1.795$$

$$\xi = -0.212$$

Using equations (12), with $\dfrac{T_o}{R^*} = \dfrac{10{,}000}{.006(\frac{2}{2})(1600)0.5} = 2060$

$$x_s = -436 \text{ ft.}$$

$$s = \text{cable length} = 3700 \text{ ft.}$$

$$z = \text{depth} = 2800 \text{ ft.}$$

The calculations may be repeated with any given set of data (19).

With the information above, the practical application of the tethering system may now be considered. The horizontal thrust force required at the submerged body is given by

$$F_h = -T_o \cos\theta_o$$

$$F_h = 9150 \text{ pounds}$$

and the thrust power required at 40 ft/sec is

$$P_h = 666 \text{ H.P.}$$

The required shaft power will depend on the thrust efficiency

$$P_s = \frac{P_h}{\eta_p}$$

Whether the practical limitation in the application of the tethering system will be the power available at the submerged body or the allowable tensile strength of cable materials is a subject for future investigation.

5. Conclusions.

For a given cable and a given set of operating conditions, there is a unique optimum cable configuration.

For heavy cables ($\frac{w}{R} > 0$), the submerged body is forward of the point where the cable emerges at the surface when the optimum condition is achieved.

The procedure used in the present calculations required that the optimum condition be determined by picking the maximum of each curve plotted in Fig. 2. If more accuracy were desired, the machine program could be altered so that this maximum is determined by the machine calculation and the remaining parameters would then be evaluated for the values θ_o, θ thus determined. For the present calculations, the additional machine time that would be required by the above procedure did not appear practical.

As shown by the sample calculation, the optimum cable configuration is

easily determined for any given cable and set of operating conditions. Also, for a given cable, auxiliary plots of z vs. U or α vs. U could be rapidly constructed.

The results of the present calculation should be valid for all cables which satisfy the assumptions used in determining the cable loading functions. The most important criterion for determining the applicability of the present calculations is that the possibility of boundary layer separations on the cable profile be minimal. Based on the above criterion, the present calculations are not applicable to circular cables or to faired cables with large thickness to chord ratios operating at Reynolds numbers for which boundary later separation may occur.

Nomenclature

C — Cable chord

C_D — Cable profile drag coefficient

F_H — Horizontal force on submerged body

P_D — Dynamic pressure $\rho_f/_2 U^2$

P_K — Thrust power

R — Hydrodynamic force acting per unit length of cable

T — Cable tension

U — Velocity of fluid relative to the cable

a — Geometric parameter defined in equation (17)

f — Normal loading function

g — Tangential loading function, or acceleration of gravity, in equation (17)

s — Length coordinate along the cable

t — Cable profile thickness

w — Weight per unit length of submerged cable element

x — Coordinate in direction of relative fluid velocity

z — Coordinate normal to relative velocity and normal to fluid surface

u — Angle between the line connecting the cable end points and the vertical (Fig. 4.)

γ_c — Specific weight of cable element

γ_f — Specific weight of fluid

θ — Angle of cable element defined in Fig. 1

θ_o — Angle of cable at submerged body (Fig. 4)

θ_s — Angle of cable at surface (Fig. 4)

η — Dimensionless depth parameter

η_p — Propulsion efficiency

ζ — Dimensionless deflection parameter

ρ_c -   Cable density

ρ_f -   Fluid density

σ -   Dimensionless length parameter

τ . -   Dimensionless tension parameter

References

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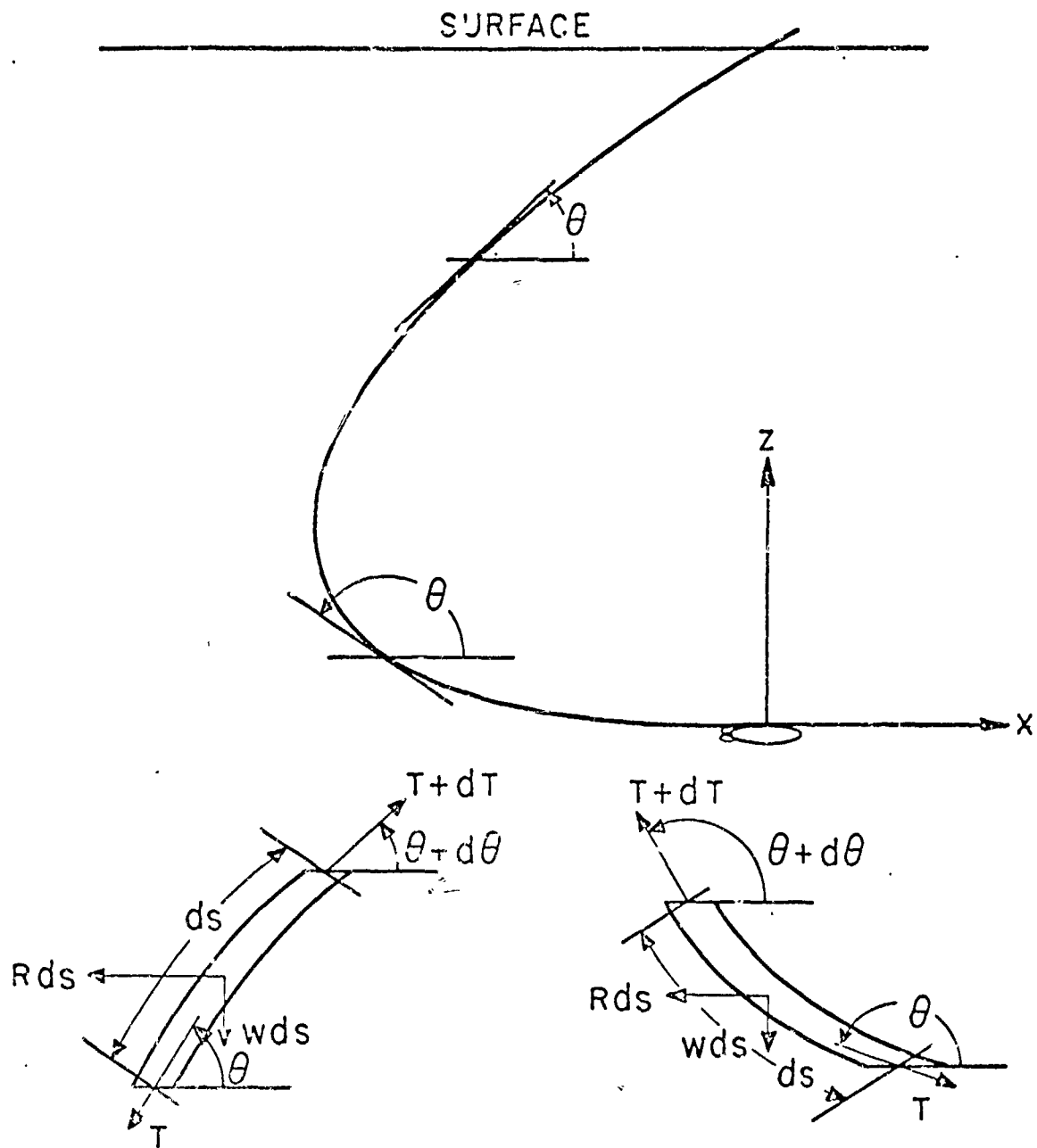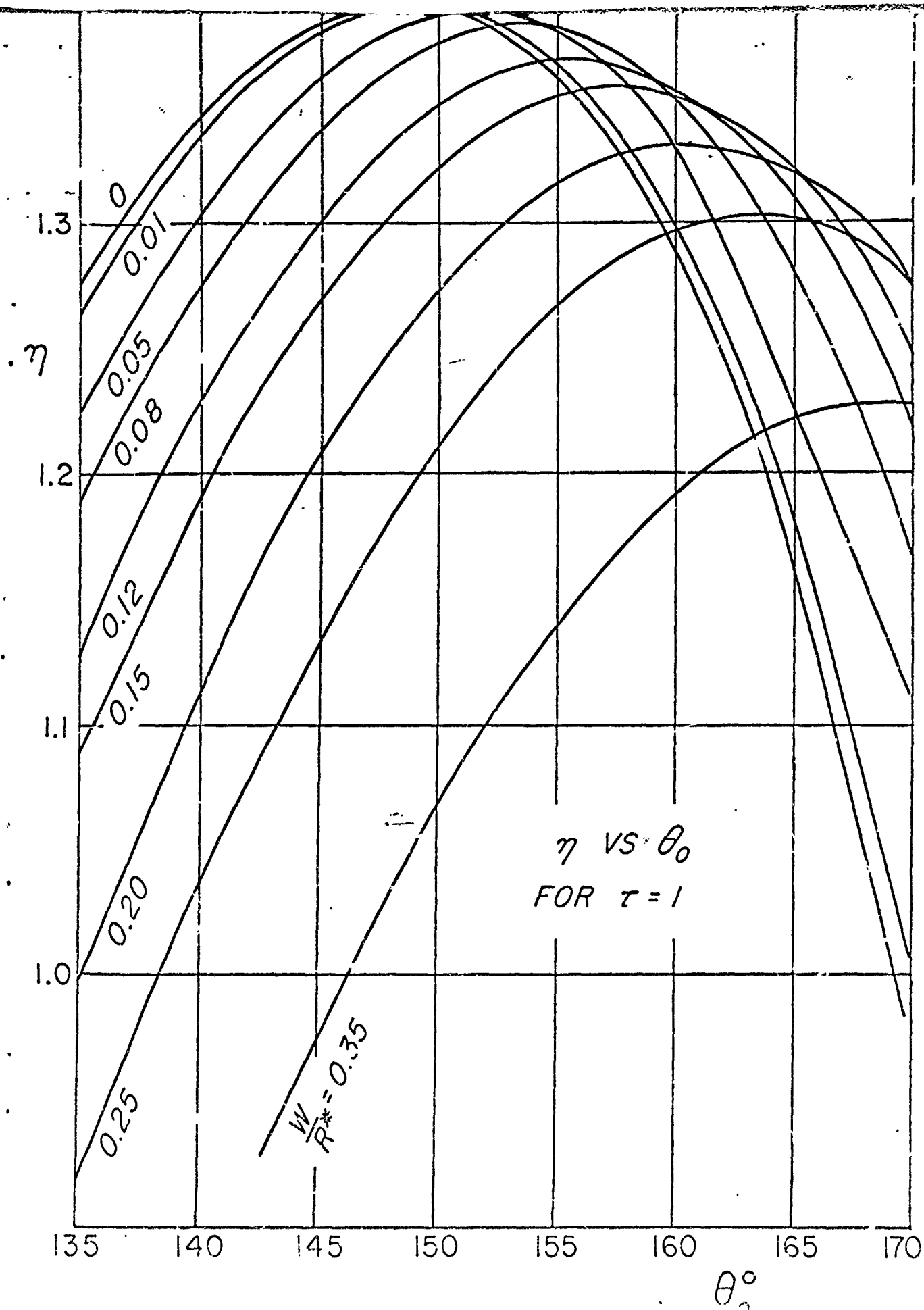   September 1963.

FIG 1 TETHERING CABLE

η VS θ_0

FOR $\tau = 1$

Curve labels: 0, 0.01, 0.05, 0.08, 0.12, 0.15, 0.20, 0.25, $\frac{W}{R^*} = 0.35$

Y-axis (η): 1.0, 1.1, 1.2, 1.3

X-axis (θ°): 135, 140, 145, 150, 155, 160, 165, 170
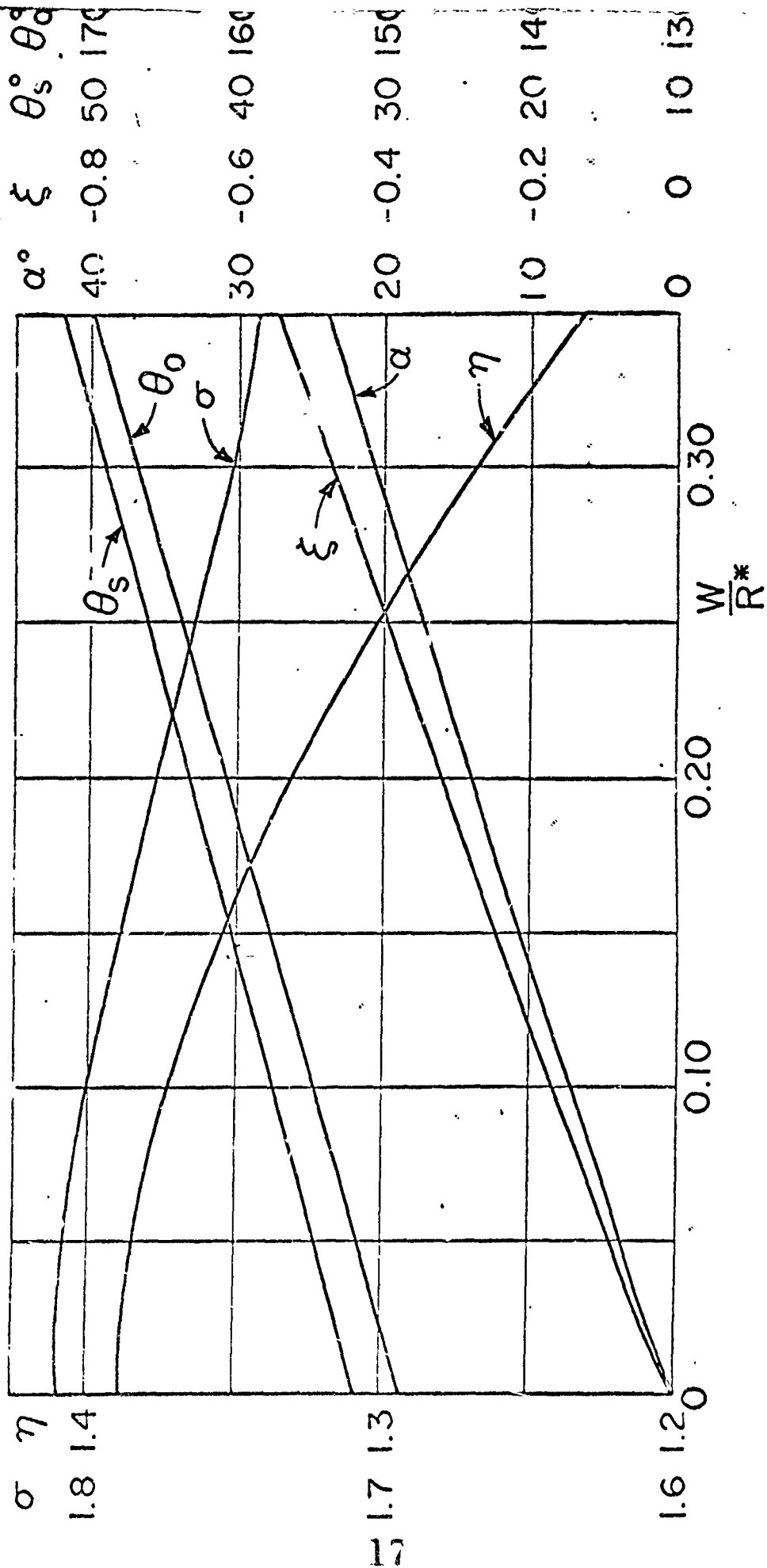
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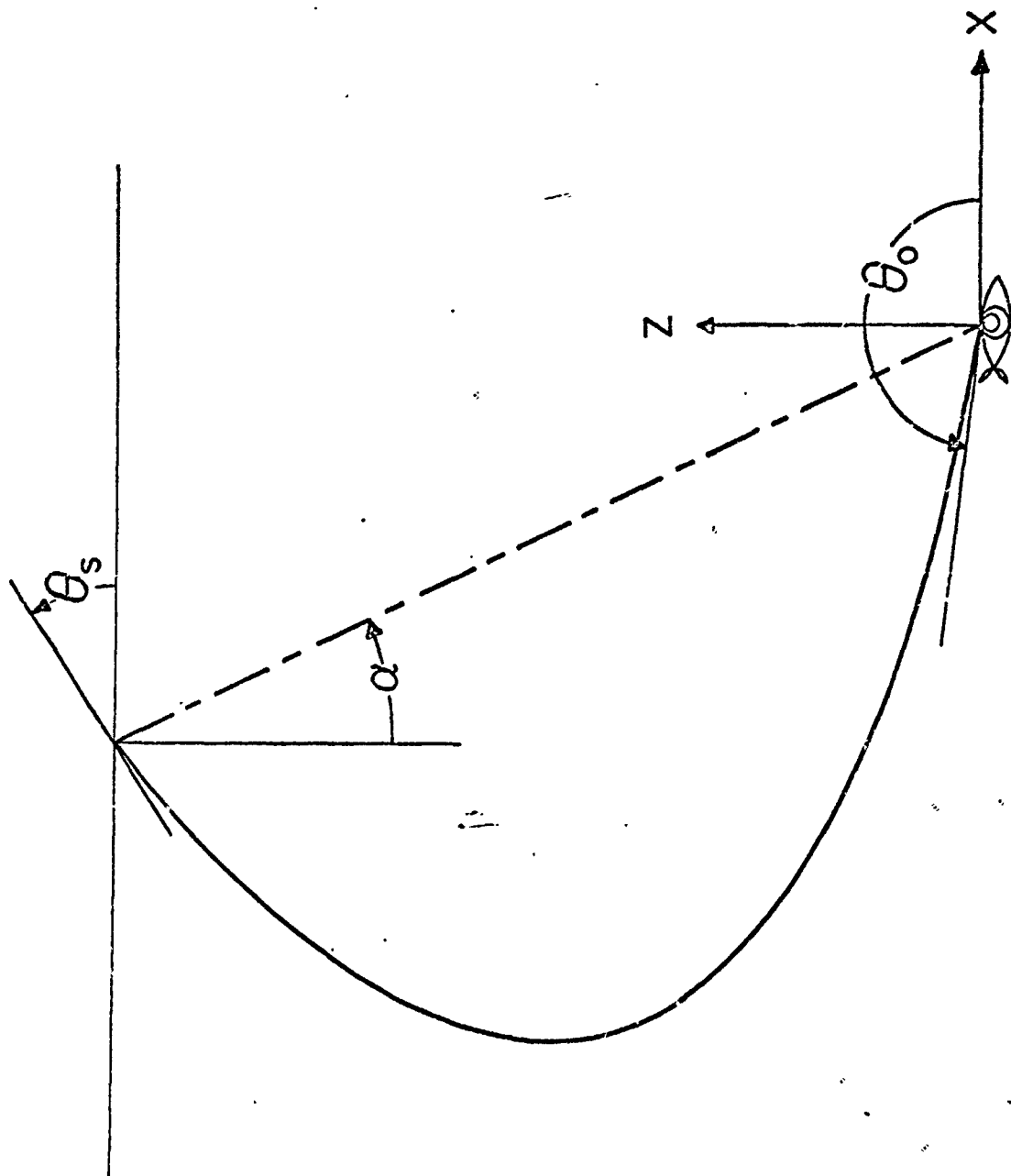CABLE PARAMETERS VS $\frac{W}{R^*}$

FOR OPTIMUM CABLE CONFIGURATION

FIG. 3

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FIG. 4 GEOMETRY OF CABLE CONFIGURATION